

# Study of Loschmidt Echo for a qubit coupled to an XY-spin chain environment

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We study the temporal evolution of a central spin-1/2 (qubit) coupled to the environment which is chosen to be a spin-1/2 transverse XY spin chain. We explore the entire phase diagram of the spin-Hamiltonian and investigate the behavior of Loschmidt echo (LE) close to critical and multicritical point (MCP). To achieve this, the qubit is coupled to the spin chain through the anisotropy term as well as one of the interaction terms. Our study reveals that the echo has a faster decay with the system size (in the short time limit) close to a MCP and also the scaling obeyed by the quasiperiod of the collapse and revival of the LE is different in comparison to that close to a QCP. We also show that even when approached along the gapless critical line, the scaling of the LE is determined by the MCP where the energy gap shows a faster decay with the system size. This claim is verified by studying the short-time and also the collapse and revival behavior of the LE at a quasicritical point on the ferromagnetic side of the MCP. We also connect our observation to the decoherence of the central spin.

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## I. INTRODUCTION

Recent years have witnessed a tremendous progress in studies of quantum information theoretic measures [1, 2] close to a quantum critical point (QCP) [3–5]. Quantities like concurrence [6–8], negativity [9, 10], quantum fidelity [11–15], quantum discord [16] etc., have been found to capture the ground state singularities associated with a quantum phase transition (QPT); for recent reviews see [17, 18].

On the other hand, the studies of decoherence namely, the quantum-classical transition by a reduction from a pure state to a mixed state have also attracted the attention of physicists in recent years [19–22]. In this connection, the concept of Loschmidt echo (LE) has been proposed to describe the hypersensitivity of the time evolution of the system to the perturbation experienced by the environment to which it is coupled [23–27]. The measure of the LE is the modulus of the overlap between two states that evolve from the same initial state  $|\psi_0\rangle$  under the influence of two Hamiltonians  $H_0$  and  $H_0 + \delta$ , where  $\delta$  is a small perturbation, given by

$$L(t) = |\langle \psi_0 | e^{i(H_0+\delta)t} e^{-iH_0t} | \psi_0 \rangle|^2.$$

In some of the recent works, attempt has been made to connect these two fields by studying the behavior of the LE close to a QCP as a probe to detect the quantum criticality. Quan *et al.* studied the decay of LE using the central spin model where a central spin-1/2 (qubit) is coupled to the environment which is chosen to be a transverse Ising chain of  $N$  spins in such a way that it is

globally coupled to all the spins of the spin chain through the transverse field term [28]. The coupling to the qubit leads to the perturbation term  $\delta$  defined above and consequently, the time evolution of the spin chain initially prepared in its ground state, gets split in two branches both evolving with the transverse Ising Hamiltonian but with different value of the transverse field. This results in the decay in the LE. It has been observed that the LE shows a sharp decay in the vicinity of the quantum critical point of the environmental spin chain; at the same time at the QCP, the LE shows collapse and revival as a function of time with the quasiperiod of revival of the LE being proportional to size of the surrounding. This study has been generalized to the case where the environment is chosen to be a transverse XY spin chain and the behavior of the LE has been studied close to the Ising critical point driven by the transverse field [29, 30].

Rossini *et al* [31], studied a generalized central spin model in which the qubit interacts with a single spin of the environmental transverse Ising spin chain and it has been shown that the decay of the LE at short time is given by the Gaussian form  $\exp(-\Gamma t^2)$  where the decay rate  $\Gamma$  depends on the symmetries of the phases around the critical point and the critical exponents. For instance, for such systems with local coupling, it has also been reported that  $\Gamma$  has a singularity in its first derivative as a function of the transverse field at the QCP [31]. In a subsequent work [32], the LE has been used as a probe to detect QPTs experimentally; at the same time, using a perturbative study in the short-time limit, the scaling relation  $\Gamma \sim (\lambda)^{-2z\nu}$  valid close to a QCP (at  $\lambda = 0$ ) has been proposed. Here,  $\nu$  and  $z$  are associated correlation length and dynamical exponents, respectively [3]. In contrast to these studies where the coupling between the qubit and the environment is chosen to be weak, it has been shown that in the limit of strong coupling the envelope of the echo becomes independent of the coupling strength which may arise due to quantum phase

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transition in the surrounding [33, 34]. Moreover the LE and the decoherence of the central spin has been studied when the environmental transverse Ising spin chain is quenched across the QCP by varying the transverse field linearly in time [35].

The central spin model we consider here, consists of a two level central spin  $S$  coupled to an environment  $E$  which is chosen to be a spin-1/2  $XY$  spin chain with anisotropic interactions and subjected to a transverse field, described by the Hamiltonian

$$H_E = - \sum_{i=1}^N [J_x \sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y + h \sigma_i^z], \quad (1)$$

where  $\sigma_i^\alpha$  ( $\alpha = x, y, z$ ) are Pauli spin matrices,  $h$  is the transverse field and  $N$  is the total number of spins in  $E$ . The spin chain (1) is exactly solvable using Jordan-Wigner mapping from spins to fermions [36–39]. The phase diagram is shown in the Fig. (1); the transition from the ferromagnetically ordered phase to the paramagnetic phase driven by the transverse field  $h$  is called the Ising transition and the transition between two ferromagnetically ordered phase, with magnetic ordering in the  $x$  direction ( $FM_x$ ) and the  $y$  direction ( $FM_y$ ), respectively, driven by the anisotropy parameter  $\gamma = J_x - J_y$ , is called the anisotropic transition. The anisotropic transition lines extend from  $h = -(J_x + J_y)$  to  $h = J_x + J_y$  along the  $\gamma = 0$  axis. Both Ising and anisotropic critical lines meet at the multicritical points as shown in the figure. We shall exploit the exact solvability of the  $XY$  spin chain to calculate the LE close to these critical points. We also note at the outset that the spin chain is to be studied under periodic boundary condition and the wave vector  $k$  takes discrete values  $k = 2\pi m/N$  with  $m = 1, 2, \dots, N/2$  and the lattice spacing is set equal to unity.

Earlier studies focussed on the case when the coupling of the central spin to the environment is through the transverse field  $h$  [28–30] and explored the behavior of LE close to the Ising critical point. Motivation behind the present work is to explore the short-time behavior, collapse and revival of the LE around the anisotropic critical point (ACP) and especially the multicritical point (MCP)  $MC_1$  of the phase diagram. To achieve this we evaluate the LE by coupling the qubit to the anisotropy term and also one of the interactions of the spin chain. Finally, we conjecture a generic scaling form that should be valid close to a QCP at least in the short-time limit.

In the next section (Sec. II), we consider the case when qubit is coupled to the anisotropy term and the behavior of the LE is explored; in Sec. III, the qubit is coupled to one of the interaction terms ( $J_x$ ). Finally in the concluding section, we discuss our results and conjecture some generic scaling relations.

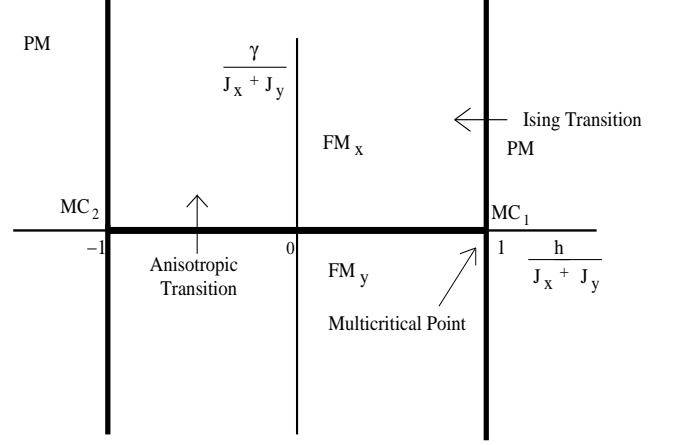


FIG. 1: The phase diagram of the anisotropic  $XY$  model in a transverse field with Hamiltonian given by (1) in the  $h/(J_x + J_y) - \gamma/(J_x + J_y)$  plane, where  $\gamma = J_x - J_y$ . The vertical bold lines denote Ising transitions from the ferromagnetic phase to the paramagnetic phase (PM), whereas the horizontal bold line stands for the anisotropic phase transition between two ferromagnetic phases  $FM_x$  and  $FM_y$ . The multicritical points at  $J_x = J_y$  and  $h = \pm 1$  are denoted by  $MC_1$  and  $MC_2$ , respectively.

## II. QUBIT COUPLED TO THE ANISOTROPY TERM OF ENVIRONMENT HAMILTONIAN

In this section, it would be useful to rewrite the Hamiltonian as

$$H_E = -\frac{1}{2} \sum_{i=1}^N [(1 + \gamma) \sigma_i^x \sigma_{i+1}^x + (1 - \gamma) \sigma_i^y \sigma_{i+1}^y + 2h \sigma_i^z], \quad (2)$$

with the choice  $J_x + J_y = 1$ , and the anisotropy parameter  $\gamma = J_x - J_y$ . Denoting the ground and excited states of the central spin by  $|g\rangle$  and  $|e\rangle$ , respectively, the coupling of the system  $S$  to the environment  $E$  can be chosen as

$$H_{SE} = -\frac{\delta}{2} |e\rangle\langle e| \sum_{i=1}^N [\sigma_i^x \sigma_{i+1}^x - \sigma_i^y \sigma_{i+1}^y], \quad (3)$$

where one assumes that the excited state of the qubit couples to all the spins of the environmental spin chain. The Hamiltonian of the composite system ( $S + E$ ) is then given by

$$H_e = H_E + H_{SE} \quad (4)$$

The form of the interaction Hamiltonian chosen in Eq. (3), enables one to analytically calculate the behavior of the LE close to the anisotropic transition line and also the MCP.

Let us assume the spin  $S$  to be initially in a pure state,  $|\phi(0)\rangle_S = c_g |g\rangle + c_e |e\rangle$ , (with coefficients satisfying  $|c_g|^2 + |c_e|^2 = 1$ ) and the environment  $E$  be in the

ground state denoted by  $|\varphi(0, \gamma)\rangle_E$ ; the total wave function of the composite system at time  $t = 0$  can then be written in the direct product form

$$|\Psi(0)\rangle = |\phi(0)\rangle_S \otimes |\varphi(0, \gamma)\rangle_E \quad (5)$$

One finds that the evolution of the  $XY$  spin chain splits into two branches (i)  $|\varphi(t, \gamma)\rangle = \exp(-iH(\gamma)t)|\varphi(0, \gamma)\rangle$  and (ii)  $|\varphi(t, \gamma + \delta)\rangle = \exp(-iH(\gamma + \delta)t)|\varphi(0, \gamma)\rangle$ ; this implies that  $|\varphi(t, \gamma)\rangle$  evolves with the Hamiltonian (2) with the anisotropy parameter  $\gamma$  and  $|\varphi(t, \gamma + \delta)\rangle$  evolves with the same Hamiltonian but the anisotropy parameter modified to  $\gamma + \delta$ . The total wave function at an instant  $t$  is then given by

$$|\Psi(t)\rangle = c_g|g\rangle \otimes |\varphi(t, \gamma)\rangle + c_e|e\rangle \otimes |\varphi(t, \gamma + \delta)\rangle. \quad (6)$$

One therefore finds the decay of the LE given by [28]

$$\begin{aligned} L(\gamma, t) &= |\langle \varphi(t, \gamma) | \varphi(t, \gamma + \delta) \rangle|^2 \\ &= |\langle \varphi(0, \gamma) | \exp(iH(\gamma)t) \exp(-iH(\gamma + \delta)t) | \varphi(0, \gamma) \rangle|^2 \\ &= |\langle \varphi(0, \gamma) | \exp(-iH(\gamma + \delta)t) | \varphi(0, \gamma) \rangle|^2, \end{aligned} \quad (7)$$

where we have used the fact that the  $|\varphi(0)\rangle$  is an eigenstate of the Hamiltonian (2) with anisotropy parameter  $\gamma$ .

The Hamiltonian (2) can be exactly solved by Jordan-Wigner (JW) transformation followed by Bogoliubov transformations [36–39] and can be written in the form  $H(\gamma + \delta) = \sum_k \varepsilon_k(\gamma + \delta)(A_k^\dagger A_k - 1/2)$  and  $H(\gamma) = \sum_k \varepsilon_k(\gamma)(B_k^\dagger B_k - 1/2)$  where  $A_k$ 's and  $B_k$ 's are Bogoliubov fermionic operators and

$$\varepsilon_k(\gamma + \delta) = \sqrt{(h + \cos k)^2 + \{(\gamma + \delta) \sin k\}^2}; \quad (8)$$

clearly,  $\varepsilon_k(\gamma) = \varepsilon_k(\gamma + \delta)$  with  $\delta = 0$ . Here we have considered periodic boundary condition, the wave vector  $k$  takes discrete values  $k = 2\pi m/N$  with  $m = 1, 2, \dots, N/2$  ( $N$  is assumed to be even and also lattice spacing is set equal to one).

In fact, under the JW transformation the Hamiltonian (2) gets reduced to direct product of decoupled  $2 \times 2$  Hamiltonians for each momentum  $k$  which in the basis  $|0\rangle$  (vacuum state) and  $|k, -k\rangle$  (two JW fermion state) can be written as

$$H_k(\gamma) = \begin{pmatrix} h + \cos k & i\gamma \sin k \\ -i\gamma \sin k & -(h + \cos k) \end{pmatrix}. \quad (9)$$

In the current problem in which the LE is calculated as a function of  $\gamma$ , one makes resort to a basis transformation to  $|\tilde{0}\rangle$  and  $|1\rangle$ , such that

$$|\tilde{0}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|k, -k\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|k, -k\rangle),$$

so that the reduced Hamiltonian (9) gets modified to

$$H_k(\gamma) = \begin{pmatrix} \gamma \sin k & h + \cos k \\ h + \cos k & -\gamma \sin k \end{pmatrix}. \quad (10)$$

The ground state of  $H(\gamma)$  and  $H(\gamma + \delta)$  can be written in the form

$$|\varphi(\gamma, 0)\rangle = \cos \frac{\theta_k(\gamma)}{2} |\tilde{0}\rangle - i \sin \frac{\theta_k(\gamma)}{2} |1\rangle$$

$$|\varphi(\gamma + \delta, 0)\rangle = \cos \frac{\theta_k(\gamma + \delta)}{2} |\tilde{0}\rangle - i \sin \frac{\theta_k(\gamma + \delta)}{2} |1\rangle,$$

where  $\tan \theta_k(\gamma + \delta) = (h + \cos k)/\{(\gamma + \delta) \sin k\}$  and  $\tan \theta_k(\gamma) = \tan \theta_k(\gamma + \delta)|_{\delta=0}$ .

The Bogoliubov operators are related to the JW operators through the relation [36]

$$A_k = \cos \frac{\theta_k(\gamma + \delta)}{2} a_k - i \sin \frac{\theta_k(\gamma + \delta)}{2} a_{-k}^\dagger. \quad (11)$$

where  $a_k$ s are the Fourier transform of the JW operators as derived through the JW transformations of spins. Using Eq. (11), one can further arrive at a relation connecting the Bogoliubov operators

$$B_k = \cos(\alpha_k) A_k - i \sin(\alpha_k) A_{-k}^\dagger \quad (12)$$

where,  $\alpha_k = [\theta_k(\gamma) - \theta_k(\gamma + \delta)]/2$

Noting the fact that  $A_k|\varphi(\gamma + \delta, 0)\rangle = 0$  and  $B_k|\varphi(\gamma, 0)\rangle = 0$  for all  $k$ , one can use the Eq. (12) to establish a connection between the ground states  $|\varphi(\gamma + \delta, 0)\rangle$  and  $|\varphi(\gamma, 0)\rangle$  given by

$$|\varphi(\gamma, 0)\rangle = \prod_{k>0} [\cos(\alpha_k) + i \sin(\alpha_k) A_k^\dagger A_{-k}^\dagger] |\varphi(0, \gamma + \delta)\rangle. \quad (13)$$

Substituting Eq. (13) to the Eq. (7), we find the expression for the LE given by

$$L(\gamma, t) = \prod_{k>0} L_k = \prod_{k>0} [1 - \sin^2(2\alpha_k) \sin^2(\varepsilon_k(\gamma + \delta)t)] \quad (14)$$

We shall use Eq. (14) to calculate the LE as a function of parameters  $\gamma$  and  $h$ , especially close the quantum critical points. As shown in Fig. (2), the LE as a function of  $\gamma$  (with the transverse field  $h < 1$ ) exhibits a sharp dip near the anisotropic critical line ( $\gamma = 0$ ). In contrary, when  $h = 1$ , and  $\gamma$  is changed, we once again observe a sharp dip near  $\gamma = 0$  which in this case happens to be the MCP  $MC_1$  ( $\gamma = 0, h = 1$ ) as shown in the phase diagram Fig. (1); changing  $\gamma$  with  $h = 1$  implies that we are in fact probing the behavior of LE along the gapless Ising critical line [40]. It should also be emphasized here that although the spin chain lies entirely on the critical line when the MCP is approached, we observe a substantial dip only at the MCP which suggests that the MCP is apparently playing the role of a dominant critical point in determining the temporal behavior of the LE [41].

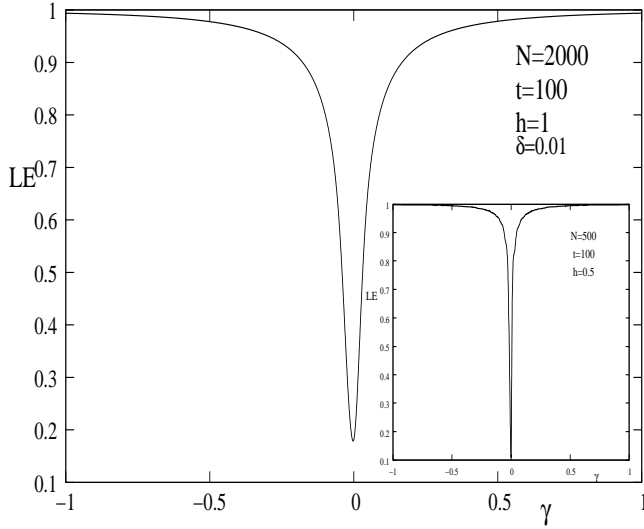


FIG. 2: The LE as a function of  $\gamma$  for  $h = 1$ ; one observes a sharp dip around the MCP ( $\gamma = -\delta$ ). Inset shows a similar dip around the anisotropic critical line (with  $h = 0.5$ ).

### A. Anisotropic Critical Point (ACP)

The correlation length exponent  $\nu$  and the dynamical exponent  $z$  associated with the ACP is the same as those of the Ising transition, i.e.,  $\nu = z = 1$  and one therefore expects that the behavior of LE should be similar to that close to the Ising transition [28]. However, one needs to consider the fact that at the ACP the energy gap vanishes at  $\gamma = 0$  for a critical mode  $k_c = \cos^{-1}(-h)$ .

As mentioned the decay of LE at short time is characterized by the critical exponents of the associated QCP. To calculate the short time behavior close to an ACP, we define a cutoff  $K_c$  such that only modes up to this cutoff are incorporated in calculating the LE [28] which is then given by  $L_c(\gamma, t) = \prod_{k>0}^{K_c} L_k$ , and one defines

$$S(\gamma, t) = \ln L_c \equiv - \sum_{k>0}^{K_c} |\ln L_k| \quad (15)$$

Expanding around the critical mode  $k_c$ , we find  $\sin^2 \varepsilon_c^k t \approx (\gamma + \delta)^2 k^2 t^2$  and  $\sin^2(2\alpha_k) \approx k^2 \delta^2 / \{\gamma^2 (\gamma + \delta)^2\}$  where we have relabeled  $k - k_c$  as  $k$ ; these lead to  $S(\gamma, t) \approx - \sum_{k>0}^{K_c} (k \delta A t)^2 / \gamma^2$ . We therefore arrive at an exponential decay of LE in the short time limit given by

$$L_c(\gamma, t) \approx \exp(-\Gamma t^2) \quad (16)$$

where,  $\Gamma = \delta^2 E(K_c) k^2 / \gamma^2$  and,  $E(K_c) = \{4\pi^2 N_c(N_c + 1)(2N_c + 1)\} / 6N^2$  (where  $N_c$  is integer nearest to  $NK_c/2\pi$ ). From above equation (16) it is clear that in this case  $L_c$  remains invariant under the transformation  $N \rightarrow N\alpha$ ,  $\delta \rightarrow \delta/\alpha$  and  $t \rightarrow t\alpha$ , with  $\alpha$  being some integer. We now proceed to study the time evolution of LE with  $h = 0.5$  and  $\gamma = -\delta$  so that

the Hamiltonian  $H((\gamma + \delta) = 0)$  is critical. We observe the collapse and revival of LE with time which is an indicator of quantum criticality as shown in fig. (3). It should be emphasized that when the size of the spin chain ( $E$ ) is doubled keeping  $\delta$  fixed, the time period of collapse and revival also gets doubled; this confirms the scaling behavior mentioned above which is also observed at the Ising critical point [28].

The quasi period of oscillations can also be calculated in the following way. From Eq. (14), we find that the mode  $k = k_c + 2\pi/N$  gives dominant contribution for  $t \rightarrow \infty$  so that for large  $N$  limit one can expand  $\varepsilon_e^k$  in the form  $\varepsilon_e^k = h + \cos k \approx \sqrt{1 - h^2} 2\pi/N$ . We have also chosen  $\gamma = -\delta$  such that  $\theta_k(\gamma + \delta) = \pi/2$  which makes

$$\sin^2 2\alpha_k = \frac{\gamma^2 \sin^2 k}{\gamma^2 \sin^2 k + (h + \cos k)^2} \approx 1. \quad (17)$$

Therefore from Eqs. (14) and (17), it is clear that oscillations in  $L(\gamma, t)$  arises due to  $\sin^2 \varepsilon_e^k t$  term providing the time period

$$T = \frac{N}{2\sqrt{1 - h^2}}. \quad (18)$$

This again shows that the time period of oscillation of the LE is proportional to the size  $N$  of the environmental spin chain as shown in the Fig. (3). Eq. (18) also shows that the time period diverges as  $h \rightarrow 1$ . This originates for the fact that for  $h = 1$ , the spin chain lies on the gapless Ising critical line, a situation which we are going to discuss in next sub-section.

We note that the decay rate  $\Gamma$  scales as  $\gamma^{-2}$  which is consistent with the scaling given in [32] since  $z\nu = 1$  for the transition across the anisotropic transition line also [39].

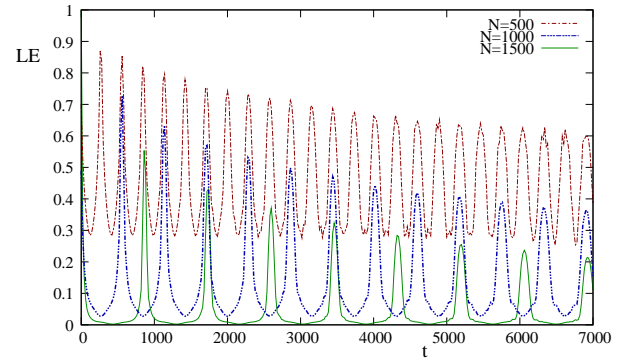


FIG. 3: The variation of the LE as a function of time at the ACP for  $\gamma = -\delta$ ,  $h = 0.5$  and  $\delta = 0.01$ . The collapse and revival of LE is indicator of a QPT and the quasiperiod of the LE is proportional to the size  $N$  of the environmental spin chain



### B. Multicritical Point (MCP)

As mentioned already, we set  $h = 1$ , and approach the MCP by changing  $\gamma$  along the Ising critical line. Expanding  $\sin \varepsilon_e^k t$  and  $\sin(2\alpha_k)$  near MCP around  $k = \pi$ ;  $\sin^2 \varepsilon_e^k t \approx (\gamma + \delta)^2 k^2 t^2$  and,  $\sin^2(2\alpha_k) \approx k^2 \delta^2 / \{4\gamma^2(\gamma + \delta)^2\}$ ;  $k \rightarrow (k - \pi)$ , one finds the short-time decay of LE given by

$$L_c(\gamma, t) \approx \exp(-\Gamma t^2) \quad (19)$$

where,  $\Gamma = \delta^2 E(K_c) / 4\gamma^2$  and,  $E(K_c) = \{(1/5)N_c^5 + (1/2)N_c^4 + (1/3)N_c^3 - (1/30)N_c\} / N^4$  (where  $N_c$  is integer nearest to  $NK_c/2\pi$ ). Equation (19) helps in providing analytical scaling for LE i.e.,  $L_c$  is invariant under transformation  $N \rightarrow N\alpha$ ,  $\delta \rightarrow \delta/\alpha^2$  and  $t \rightarrow t\alpha^2$ . This scaling has to be contrasted with the scaling of  $L_c$  close to the ACP presented in the previous section. We note that at the MCP, the minimum energy gap scales as  $(k - \pi)^2$  so that  $z = 2$  whereas near an ACP, it scales linearly as  $(k - k_c)$  with  $z = 1$ . This difference in the dynamical exponent is the reason behind different scaling observed in the short-time limit. The collapse and revival of LE as a function of time is shown in Fig. (4) for different system sizes and fixed  $\delta$ ; this confirms the scaling observed in the short-time limit. At the same time, we note that  $\Gamma \sim \gamma^{-2}$  as the exponent  $z\nu = 1$ , even for transition across the MCP.

**To calculate the time-period of oscillation, we again proceed using the same line of arguments given in section A. At the MCP ( $h=1$ ),  $\varepsilon_e^k = h + \cos k \approx 2\pi^2/N^2$  and similarly for  $\gamma = -\delta$ , it can be shown that  $\sin^2 2\alpha_k \approx 1$ . The time period of oscillations in  $L(\gamma, t)$  is therefore given by  $T \approx N^2/2\pi$ , which confirms that the LE oscillates with period is proportional to  $N^2$  at MCP (see Fig. (4)).**

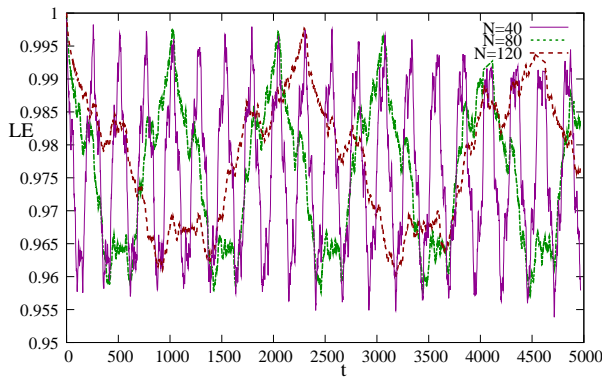


FIG. 4: The LE is shown as a function of time at the MCP  $\gamma = -\delta$ ,  $h = 1$  and  $\delta = 0.01$ . The quasiperiod of the collapse and revival is proportional to  $N^2$  according to the scaling relation  $N \rightarrow N\alpha$ ,  $\delta \rightarrow \delta/\alpha^2$  and  $t \rightarrow t\alpha^2$ , discussed in the text. We note that the collapse and revival close to the MCP is not a smooth function of time.

### III. QUBIT COUPLED TO THE INTERACTION TERM OF THE ENVIRONMENT HAMILTONIAN

In this section we shall choose the form of the  $XY$  Hamiltonian given in Eq. (1); transforming to a new state of basis vectors defined by [42]

$$|e_{1k}\rangle = \sin(k/2)|0\rangle + i \cos(k/2)|k, -k\rangle$$

$$|e_{2k}\rangle = \cos(k/2)|0\rangle - i \sin(k/2)|k, -k\rangle,$$

one can rewrite the reduce  $2 \times 2$  Hamiltonian (9)  $H_k$  in the form

$$\begin{pmatrix} J_x + J_y \cos 2k + h \cos k & J_y \sin 2k + h \sin k \\ J_y \sin 2k + h \sin k & -(J_x + J_y \cos 2k + h \cos k) \end{pmatrix}. \quad (20)$$

We choose the coupling term given by

$$H_{SE} = -\delta|e\rangle\langle e| \sum_{i=1}^N [\sigma_i^x \sigma_{i+1}^x], \quad (21)$$

Therefore total Hamiltonian (spin and environment) becomes

$$H_e = - \sum_{i=1}^N [(J_x + \delta|e\rangle\langle e|)\sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y + h \sigma_i^z], \quad (22)$$

The advantage of selecting such a coupling is that it enables us to explore the MCP and ACP via different paths and compare the results with the previous case, e.g., if one chooses  $J_x = 2h_y$ , the MCP is approached along a linear path when  $J_x$  is changed unlike the previous case when it is approached along the Ising critical line.

Following identical mathematical steps as described in the previous section, one can find that the expression of the LE is given by

$$L(J_x, t) = \prod_{k>0} L_k = \prod_{k>0} [1 - \sin^2(2\alpha_k) \sin^2(\varepsilon_k(J_x + \delta)t)] \quad (23)$$

where,  $\alpha_k = [\theta_k(J_x) - \theta_k(J_x + \delta)]/2$ , and

$$\theta_k(J_x + \delta) = \arctan \left[ \frac{J_y \sin(2k) + h \sin(k)}{J_x + \delta + J_y \cos(2k) + h \cos(k)} \right], \quad (24)$$

The energy spectrum given in Eq. (8) can be rewritten as

$$\varepsilon_k(J_x + \delta) = [(J_y \sin 2k + h \sin k)^2 + (J_x + \delta + J_y \cos 2k + h \cos k)^2]^{1/2}. \quad (25)$$

Let us first explore the LE close to different critical points; refereeing to fig (5) and (6), we find that there is a sharp dip in LE wherever the parameters values are such that the system is close to a critical point. For example, in Fig. (5), we have varied  $J_x$  keeping  $h$  and  $J_y$  fixed and  $h < 2J_y (= 2)$  such that we observe dips at two

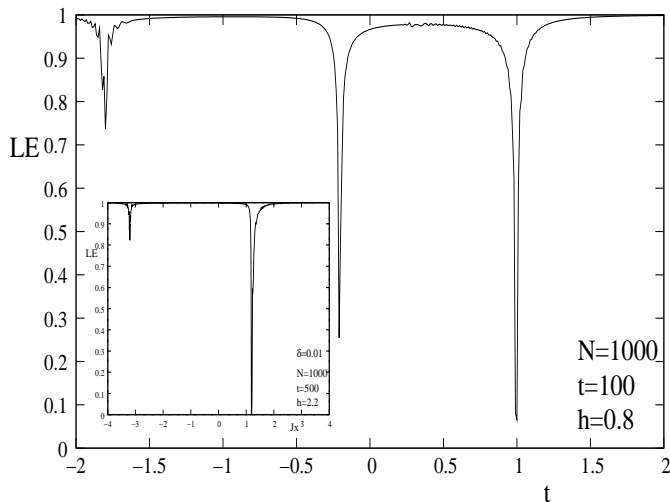


FIG. 5: The LE plotted as a function of  $J_x$  shows dips around the Ising critical points ( $J_x = -0.2, -1.8$ ) as well as the ACP ( $J_x = 1$ ) with  $J_y = 1$  and  $h = 0.8 (< 2J_y)$ . Inset shows that for  $h = 2.2 (> 2J_y)$ , there are dips only at the Ising critical points as the variation of  $J_x$  does not take the spin chain across the anisotropic critical line.

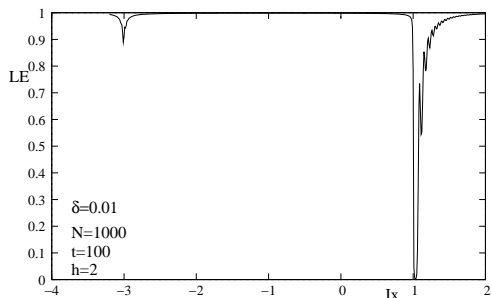


FIG. 6: The interaction  $J_x$  is varied with  $h$  fixed to  $h = 2J_y = 2$ . The LE shows a dip at Ising critical point ( $J_x = -3$ ) and also at the MCP ( $J_x = 1$ ).

Ising critical points and also at the anisotropic critical point; for  $h > 2J_y$ , in contrast, one observes dips only at the Ising critical points as the anisotropic transition point is not crossed in the process of changing  $J_x$ . For  $h = 2J_y$ , one observes dips at the Ising critical point and the MCP as  $J_x$  is varied (Fig. (6)). Equipped with these observations, we now proceed to study the short time decay of LE close to these critical points.

### A. Short time behavior

Near the critical points and the MCP, we have identical short-time behavior and scaling with respect to  $N$ ,  $\delta$  and  $t$  as already reported in the previous section. These are corroborated by numerical estimation of collapse and revival close to the critical and the multicritical point as

shown in Figs. (7) and (8). This confirms that the scaling of LE does not depend on how the central spin is coupled to the environment rather it is hypersensitive to the proximity to a critical point of the environment.

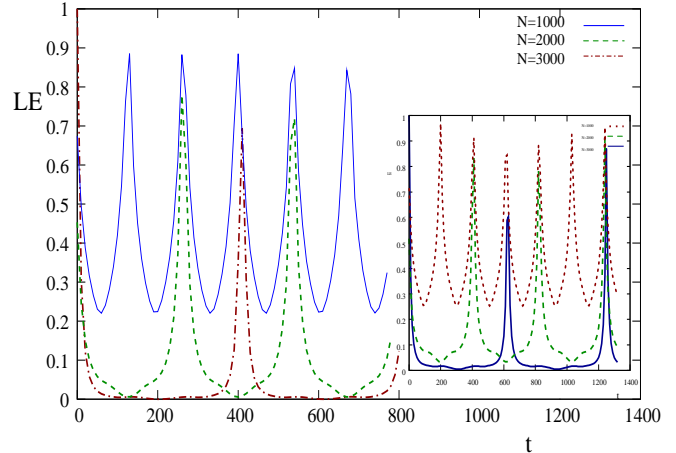


FIG. 7: The collapse and revival of the LE at the ACP ( $J_x = 1 - \delta$ ,  $h = 0.8, J_y = 1$ ). The inset shows the same behaviour at the Ising critical point ( $h = 2.2, J_x = 1 - \delta$  and  $J_y = 1$ ). In both the cases  $\delta = 0.01$

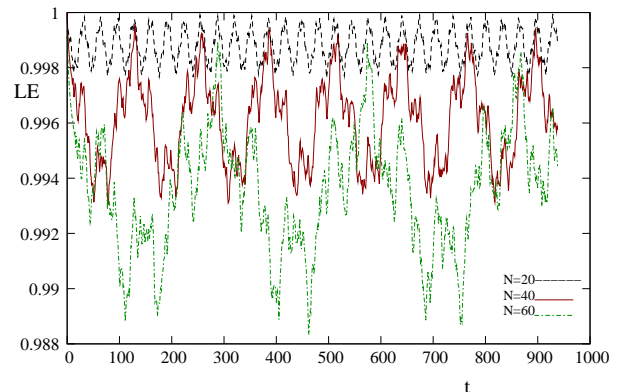


FIG. 8: The time variation of LE at MCP ( $J_x = 1 - \delta, h = 2 = 2J_y$ ) is shown. The quasiperiod of LE is again proportional to  $N^2$  as reported in Sec. II.

### B. Close to the MCP

It is well known that for a finite XY spin chain, there exist quascritical points on the ferromagnetic side close to the MCP; the energy gap is locally minimum at these quascritical points and it scales  $k^3$  in contrast to the scaling  $k^2$  at the MCP. In the limit of  $N \rightarrow \infty$ , all these quascritical points approach the MCP. These quascritical points and exponents associated with them have been found to dictate the scaling of the defect den-

sity following a slow quench across the MCP [41, 43] and also the scaling of fidelity susceptibility close to it [44]. We shall now explore the collapse and revival of the LE fixing the parameters such that  $J_x + \delta$  is right at a quasicritical point. For modes  $k \approx \pi$ , one can use the simplification,  $\sin^2 \varepsilon_e^k t \approx (J_x + \delta - J_y)^2 t^2$  and,  $\sin^2(2\alpha_k) \approx 4J_y^2 k^6 \delta^2 / \{(J_x - J_y)^2 (J_x + \delta - J_y)^2\}$ . We therefore get a similar exponential decay of the LE  $L_c(J_x, t) \approx \exp(-\Gamma t^2)$  with

$$\Gamma = \frac{4J_y^2 \delta^2 E(K_c)}{(J_x - J_y)^2} \quad \text{and} \quad E(K_c) = \frac{A(N_c)}{N^6}, \quad (26)$$

where  $A(N_c) = (1/7)N_c^7 + (1/2)N_c^6 + (1/2)N_c^5 - (1/6)N_c^3 + (1/42)N_c$ , and as defined previously,  $N_c$  is integer nearest to  $NK_c/2\pi$ . The above equation (26), shows a very interesting scaling behavior of the LE  $N \rightarrow N\alpha$ ,  $\delta \rightarrow \delta/\alpha^3$  and  $t \rightarrow t\alpha^3$  which is different from the scaling observed at the MCP. **Similar to previous cases, at the quasicritical point  $\varepsilon_e^k = h + \cos k \approx 16\pi^3/3N^3$  with  $h = 2J_y = 2$ ; for  $J_x = 1 - \delta + 4\pi^2/N^2$  and large  $N$ , it can be easily shown that  $\sin^2 2\alpha_k \approx 1$ . The time period of oscillations in  $L(J_x, t)$  is therefore given by  $T \approx N^3/16\pi^2$ , which verifies the fact that LE oscillates with period proportional to  $N^3$  at quasi critical point (as shown in Fig. (9)).**

The collapse and revival of the LE as a function of time supports the scaling behavior analytically obtained in the short time limit (see Fig. (9)). Comparing Eq. (26) with the form of decay rate  $\Gamma$  given in Eq. (16), we find that in both the cases  $\Gamma \sim 1/\gamma^2$ ; this is because at a quasicritical point one can define an effective dynamical exponent  $z_{qc} = 3$   $\nu_{qc} = 1/3$  such that  $\nu_{qc} z_{qc} = 1$  [41, 43]. Moreover, we find that the quasiperiod scales as  $N^{z_{qc}}$ .

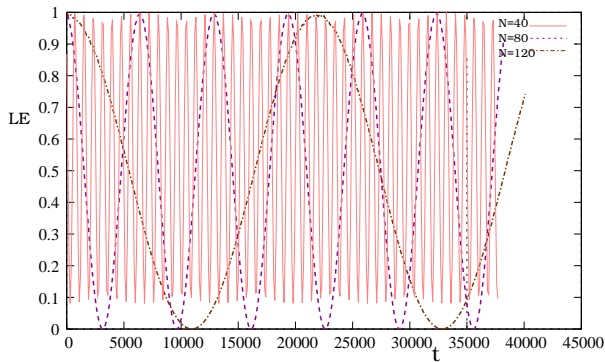


FIG. 9: Collapse and revival of the LE at a quasicritical point ( $J_x = 1 - \delta + 4\pi^2/N^2$ ,  $h = 2$  and  $J_y = 1$ ) as defined in the text. The quasiperiod scales with  $N^3$  in contrast to  $\sim N^2$  at the MCP. This is consistent with the scaling  $N \rightarrow N\alpha$ ,  $\delta \rightarrow \delta/\alpha^3$  and  $t \rightarrow t\alpha^3$ .

## IV. CONCLUSION

In this paper, a spin-1/2 (qubit) is coupled to the environment which is chosen to be a spin-1/2  $XY$  spin chain and the temporal behavior of the LE is studied. The coupling is done in such a way that enables us to study the LE close to the ACP as well as the MCP of the phase diagram and these points are approached in different fashions, e.g., the MCP is approached along the Ising critical line in Sec. II while in Sec. III it is approached following a linear path. We find that close to the ACP, the evolution of the LE is identical to that reported in the ref. [28]. However, around the MCP, we observe that the quasiperiod of the collapse and revival of the LE as a function of time scales as  $N^2$  where  $N$  is the size of the environmental spin chain. We attribute this to the fact that the dynamical exponent  $z$  associated with the MCP is two. To justify this conjecture, we have estimated the scaling of the decay rate  $\Gamma$  and also the period of the collapse and revival of the LE at a quasi-critical point on the ferromagnetic side of the MCP. We find that quasiperiod scales as  $N^3$ . It should be noted here that at the quasicritical point, the minimum gap scales with the system size as  $N^3$  and hence one can define an equivalent dynamical exponent  $z_{qc} = 3$  [43]. In Sec. II, even though the MCP is approached along a gapless critical line, a sharp dip in the LE is observed only around the MCP where the decay of energy gap with the system size is faster ( $\sim 1/N^2$ ) with respect to that near the Ising or anisotropic critical point. We observe that the collapse and revival of the LE at MCP is not a smooth function of time which is attributed to the fact that in Sec II the spin chain is always close to the Ising critical line whereas in Sec. III quasicritical points are likely to influence the temporal evolution of the LE. These quasi-critical points, on the other hand, are expected to be related to the proximity to the critical line of the finite-momentum anisotropic transition.

Although we have studied an integrable spin chain reducible to direct product of two-level system, our studies indicate the possibility of some interesting scaling behavior. We see that in all the cases studied here, the LE decays exponentially close to the critical point in the short time limit with the decay rate  $\Gamma$  scaling as  $\Gamma \sim \lambda^{-2z\nu}$  i.e., our studies support the scaling proposed in [32] based on perturbative calculations and a Landau-Zener argument. Moreover, we find the quasiperiod of the collapse and revival of the LE at the critical point scales as  $N^z$ ; we note that the dynamical exponent  $z$  determines how does the minimum energy gap vanishes with increasing system size ( $\sim N^{-z}$ ) at the QCP. At a quasicritical point the effective dynamical exponent  $z_{qc} = 3$  is found to determine the scaling of the quasiperiod of collapse and revival with the system size.

Finally, we comment on the decoherence of the central spin during time evolution which is calculated using its reduced density matrix [28]. The off-diagonal terms of the reduced density matrix is given by  $c_g^* c_e d(t)$  and its

hermitian conjugate where the decoherence factor  $d(t)$  is connected to the LE through the relation  $L(t) = |d(t)|^2$  [35]. The vanishing of the LE around to the QCP therefore implies a complete loss of coherence and therefore the qubit makes transition to a mixed state even though initial state is chosen to be pure. On the other hand, away from the QCP LE stays close to unity, thus the purity of the qubit state is retained. Our studies reveal that close to the MCP,  $\Gamma \sim 1/N^2$  in the short time limit, implying a faster loss of coherence with the increasing system size when the environment  $E$  is close to a MCP

than when it is close to a QCP. The loss is even faster when the spin chain sits at a quasicritical point close to the MCP. This faster loss of coherence with the system size, we believe, is a note-worthy observation.

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